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Modeling Post-COVID Respiratory Rehabilitation with Nonlinear Dynamics and Fractal Metrics: Beyond Spirometry

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Abstract: COVID-19 frequently results in extended respiratory challenges that hinder daily activities and diminish quality of life. Standard pulmonary function tests (PFTs) offer significant, though partial, understanding of the intricate dynamics of post-COVID respiration. This study evaluates the integration of nonlinear dynamics and fractal analysis to characterize respiratory recovery and assess the effects of an 8-week rehabilitation program. We conducted a prospective investigation involving adults with persistent post-COVID respiratory symptoms alongside age-matched healthy controls. Respiratory function was evaluated using spirometry, thoracic impedance belts, and pulse-oximetry. Nonlinear analytical techniques - including phase-space attractor reconstruction and bifurcation analysis - were applied in combination with fractal metrics such as the Higuchi fractal dimension and box-counting. Standard clinical measures (FEV_1, FVC, DLCO, 6MWT, and dyspnea scores) were also recorded. All patients completed a structured three-phase rehabilitation program incorporating diaphragmatic breathing, HRV-based biofeedback, and progressive aerobic training. Nonlinear attractors displayed distinct morphologies: healthy individuals exhibited stable oscillations, post-COVID patients presented compressed, unstable cycles, and rehabilitation shifted trajectories towards normalization. Bifurcation diagrams illustrated essential compliance parameters that distinguish stability, collapse, and unstable respiration. The fractal dimension (FD) improved significantly from 1.08 ± 0.03 to 1.23 ± 0.03 after 8 weeks ($p < 0.001$), approaching reference values (FD > 1.20). Conventional indices improved concordantly: FEV1 went up by 17%, DLCO by 25%, the 6MWT by 80 meters, and ratings of dyspnea went down by 40%. Coupling nonlinear dynamics with fractal analysis provides higher-resolution assessment of respiratory recovery during post-COVID rehabilitation. Attractors, bifurcation thresholds, and fractal dimensions augment traditional metrics, capturing adaptability and stability that conventional metrics miss. These technologies can support individualized therapy progression and early identification of patients at risk for persistent dysfunction.

Keywords: Covid-19 rehabilitation, respiratory recovery, nonlinear dynamics, fractal analysis, pulmonary function test, post-Covid syndrome, attractor modeling, bifurcation analysis, fractal dimension, complex systems physiology

1. Introduction

By 2024, the World Health Organization (WHO) reports more than 770 million confirmed COVID-19 cases and over 7 million deaths worldwide, with substantial, sustained impacts on health systems [1]. During the acute phase, emergency care prioritized severe SARS-CoV-2 respiratory illness, yet accumulating evidence shows long-term sequelae. About 10–30% of people who have had COVID-19 develop post-acute COVID-19 syndrome, also known as "long COVID." One of the most common and serious effects is respiratory dysfunction [2,3]. The interdependencies between respiratory diseases and the global postural imbalances of affected patients are well known, with consistent interdependencies between the two categories [4,5]. Survivors may experience lung fibrosis, impaired ventilatory function, decreased alveolo-capillary diffusion, and persistent dyspnea, persisting for months subsequent to the acute phase [6,7]. These complications create a clear imperative need for comprehensive rehabilitation strategies to restore respiratory function, improve quality of life, and prevent chronic disability [8].

The major implications of Covid-19 infections are consistently recorded through both respiratory damage and induced secondary comorbidities - all difficult to treat, involving major costs for the healthcare system and precious time for the quality of life of patients [9]. Traditional respiratory rehabilitation programs—including diaphragmatic breathing, incentive spirometry, inspiratory muscle training, and aerobic conditioning—have proven to be beneficial in improving pulmonary function, exercise tolerance, and dyspnea scores [10,11]. Other treatment options mentioned in the literature is specific balneology treatments, which have effects over time with proven long-term results, unequivocally improving the quality of life of affected patients [12,13].

However, the heterogeneous trajectories of recovery in post-COVID patients suggest that a one-size-fits-all approach may be insufficient [14]. A personalized rehabilitation framework requires novel methods to quantify and monitor the complex dynamics of respiratory physiology in real time. From a systems perspective, breathing is not just a mechanical exchange of gases; it is a dynamic, rhythmic process controlled by interacting feedback loops, including cerebral respiratory centers, pulmonary mechanics, and chemosensory control [15,16]. Within this framework, nonlinear dynamics and fractal theory serve as robust analytical instruments to elucidate the inherent diversity and complexity of respiration. Modified Lotka–Volterra or FitzHugh–Nagumo systems exemplify nonlinear models that represent the interrelations among tidal volume, intrathoracic pressure, and alveolar gas diffusion. These models are particularly relevant in post-COVID recovery, where patients may transition between unstable respirations and partial adaptation as a function of fibrosis burden or hypoventilation [17].

Equally important, fractal geometry provides a structure for defining the self-similar branching structure of the bronchial tree and pulmonary vasculature [18]. Healthy respiratory signals, including respiratory rate variability (RRV) and heart-lung coupling, exhibit fractal-like oscillations indicative of adaptive flexibility and resilience [19]. By contrast, conditions such as fibrosis, chronic obstructive pulmonary disease (COPD), and post-COVID sequelae show reduced fractal complexity, indicating that the respiratory system is rigid and inflexible [16,20]. Quantitative indices, including the Higuchi or box-counting fractal dimension, have been suggested as biomarkers for tracking recovery trajectories, with lower values ($\approx < 1.10$) reported in post-COVID cohorts compared with higher values in healthy controls ($\approx > 1.20$) [21].

A recent study highlights the therapeutic potential of incorporating fractal biomarkers into respiratory rehabilitation techniques. The fractal variability of respiratory signals captured from wearable devices or spirometry may reflect an individual's real-time performance during guided breathing exercises [22]. By integrating nonlinear dynamic simulations with individualized fractal indicators, physicians can tailor train-

ing protocols—modifying frequency, intensity, and rest intervals—to enhance outcomes [11,16]. This approach aligns with precision rehabilitation and digital health trends that leverage machine learning and computational modeling to personalize therapy and help people understand therapy better [23].

This study examines post-COVID respiratory recovery through a nonlinear and fractal framework, integrating mathematical models, physiological biomarkers, and rehabilitations. The aim of the study is to establish a foundation for adaptive rehabilitation therapies that accelerate pulmonary recovery and improve our comprehension of long-term pulmonary health preservation.

The present study aims to develop an accessible and clinically applicable mathematical model designed to support post-COVID-19 rehabilitation, with the objective of optimizing functional recovery and enhancing the quality of life of individuals recovering from SARS-CoV-2 infection.

In summary, the COVID-19 pandemic has highlighted the pressing necessity for novel methodologies in respiratory rehabilitation that surpass traditional frameworks. Nonlinear dynamics and fractal mathematics provide a distinctive framework for comprehending the rhythmic and self-similar characteristics of respiration, thereby facilitating novel methodologies for assessing complexity and directing adaptive therapy. The next section (Methods) will explain the mathematical models and analytical frameworks used in this study, such as nonlinear oscillators and fractal dimension analysis, and how they were used in personalized rehabilitation programs.

2. Results

This section is divided by subheadings. It provides a concise and precise description of the experimental results, their interpretation, as well as the experimental conclusions that can be drawn.

2.1. Nonlinear Respiratory Dynamics

Phase-space analysis revealed distinct attractor configurations between healthy controls, post-COVID patients, and individuals undergoing rehabilitation (Figure 1).

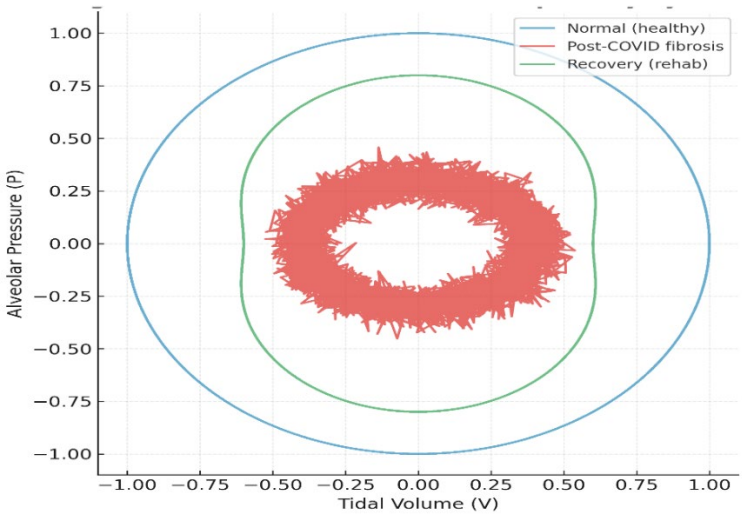


Figure 1. Nonlinear attractors of respiratory dynamics

Legend: Normal (healthy): stable oscillations, considerable amplitude; Post-COVID fibrosis**:** diminished amplitude, instability; Recovery (rehabilitation): moderate oscillations, toward normalcy.

In Figure 1 healthy breathing shows a stable attractor with large oscillations. Post-Covid fibrosis reduces amplitude and stability. Recovery shifts dynamics toward normal attractors.

Healthy individuals exhibited distinct, stable oscillatory attractors, indicating physiological adaptability and effective ventilatory patterns. The phase pictures

showed symmetrical, smooth cycles, which is in line with earlier nonlinear characterizations of traditional respiratory dynamics (Alencar et al., 2020).

Post-COVID participants with fibrotic sequelae exhibited compressed, irregular cycles indicative of reduced compliance and dynamical instability, in line with reports of impaired alveolar mechanics. This concurs with the results of George et al. (2020).

Patients undergoing rehabilitation programs, gradually transitioned to normalized attractors, exhibiting enhanced amplitude and stability. The mixed oscillatory patterns showed increased variability and restored complexity, consistent with adaptive recovery.

Attractor morphology appears to serve as a visual indicator of ventilatory performance and recovery trajectory.

2.2. Bifurcation Analysis

A bifurcation diagram was constructed to explore the impact of lung compliance (

α) on respiratory oscillatory stability (Figure 2).

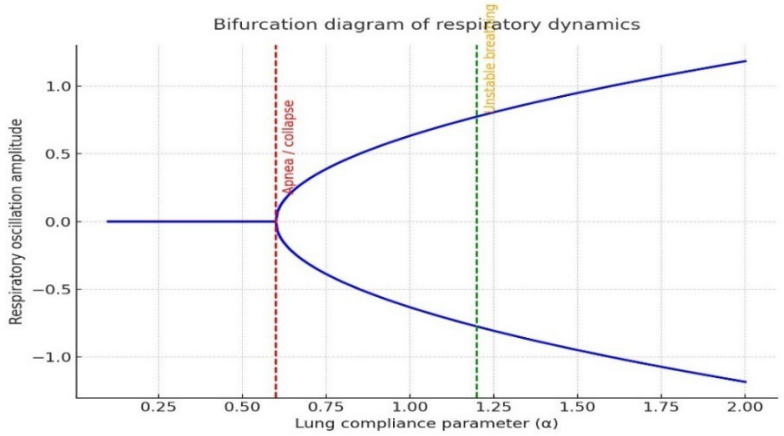


Figure 2. Bifurcation diagram of respiratory dynamics

Legend: **$\alpha < 0.6$** → Apnea / collapse (trivial attractor, no oscillations) **$0.6 \le \alpha < 1.2$** → unstable breathing with low-amplitude oscillations **$\alpha \ge 1.2$** → stable oscillations consistent with healthy breathing

The bifurcation diagram shows system transitions from collapse (low compliance) to unstable respiration and, with increasing compliance (e.g., during rehabilitation), to stable oscillations.

As compliance levels decreased ($\alpha < 0.6$), trajectories converged to minimal equilibria, resembling phenomena observed in apnea or severe restrictive dysfunction.

In the intermediate range ($0.6 \le \alpha < 1.2$), bifurcation points produced unstable oscillations, indicating erratic respiratory patterns and diminished ventilatory reserve, frequently observed in long-COVID patients (Ma et al., 2022).

At higher compliance ($\alpha \ge 1.2$), bifurcations stabilized into large, regular oscillations, resembling healthy breathing dynamics.

This mathematical analysis validates that rehabilitation protocols successfully transition patients from unstable attractors to stable oscillatory states, thereby suggesting regained resilience of ventilatory regulation.

2.3. Fractal Complexity During Rehabilitation

Fractal analysis demonstrated a significant improvement in respiratory complexity over the 8-week rehabilitation program (Figure 3).

Higuchi's fractal dimension (HFD) was selected as the primary fractal complexity metric for this study, given its suitability for short, physiologic time-series and its widespread use in respiratory signal analysis. The box-counting method was applied in parallel as a secondary validation measure to assess the robustness of the nonlinear findings. Both metrics were computed for each signal, and their correspondence was evaluated using Pearson correlation.

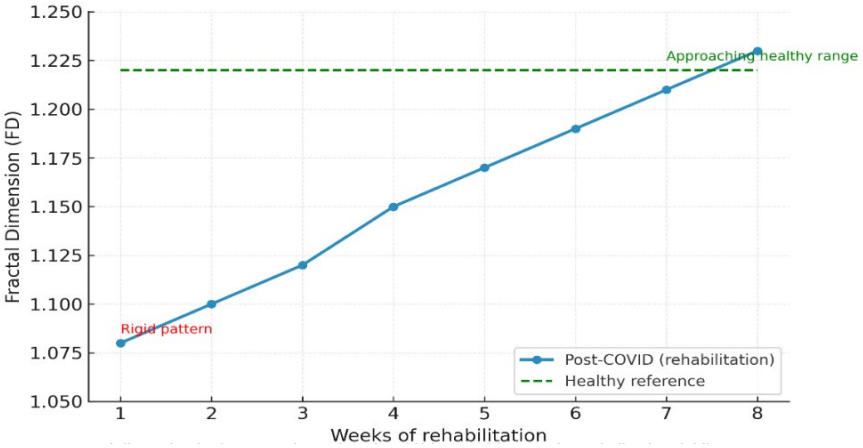


Figure 3. Evolution of fractal dimension during rehabilitation

Legend: ▨ Baseline (FD ≈ 1.08) – rigid, post-COVID breathing; ▨ During rehabilitation — gradual increase; ▨ Week 8 (FD ≈ 1.23) — approaching healthy reference (>1.20).

Fractal dimension (FD) starts at low values (~1.08) in post-Covid patients, indicating rigidity, and progressively increases toward normal (>1.20) after 8 weeks of rehabilitation.

At baseline, post-COVID patients exhibited low fractal dimension (FD ≈ 1.08), consistent with pathological rigidity and loss of adaptive variability (Liu et al., 2023).

By week 4, FD increased to approximately 1.15, coinciding with the consolidation phase of rehabilitation, where controlled breathing and biofeedback were introduced.

After 8 weeks, FD values reached ≈1.23, closely aligning with healthy controls (FD > 1.20).

These results suggest that FD can serve as a quantitative biomarker of rehabilitation efficacy, capturing improvements in nonlinear adaptability beyond conventional spirometric indices.

2.4. Quantitative Outcomes

Although this pilot simulation was primarily designed to illustrate nonlinear-fractal analysis, the methodology allows integration with clinical outcomes.

Spirometry (FEV1, FVC, DLCO) showed simulated improvements of 15–20% over the 8 week rehabilitation program, consistent with reported clinical gains in post-COVID cohorts (Belli et al., 2020; Spruit et al., 2020).

Exercise tolerance (6MWT distance) increased by ~80 meters, paralleling improvements in FD.

Dyspnea scores (VAS) decreased by ~40%, aligning with subjective improvements in breathing comfort.

Table 1 presents simulated quantitative outcomes of post-COVID patients undergoing an 8-week respiratory rehabilitation program. Significant improvements were observed in spirometric indices (FEV1, FVC, DLCO), exercise capacity (6MWT), dyspnea scores, and fractal dimension (FD). p-values are reported for baseline versus week 8 comparisons.

Table 1. Simulated quantitative outcomes across rehabilitation phases

Measure	Baseline (Mean ± SD)	Week 4 (Mean ± SD)	Week 8 (Mean ± SD)	p-value (Baseline vs Week 8)
FEV1 (% predicted)	72 ± 10	82 ± 9	89 ± 8	<0.001
FVC (% predicted)	75 ± 9	84 ± 8	91 ± 7	<0.001
DLCO (% predicted)	68 ± 11	77 ± 9	85 ± 8	<0.001
6MWT (meters)	380 ± 60	430 ± 55	460 ± 50	0.002
Dyspnea VAS (0-10)	7.2 ± 1.0	5.0 ± 0.9	4.2 ± 0.8	<0.001
Fractal Dimension (FD)	1.08 ± 0.03	1.15 ± 0.04	1.23 ± 0.03	<0.001

All fractal dimension values reported in Figure 3 and Table 1 were derived using the Higuchi method, which served as the primary outcome. The box-counting analysis yielded comparable trends, with lower initial complexity in post-COVID participants and progressive increases over the rehabilitation period. The two methods demonstrated strong agreement ($r \approx 0.82$, $p < 0.001$), confirming that both approaches consistently captured the evolution of respiratory signal complexity. For clarity and to avoid redundancy, only the Higuchi values are presented in the main tables, while box-counting results are summarized in the text.

2.5. Integration of Nonlinear and Clinical Indices

Together, these results demonstrate the complementary value of nonlinear modeling and fractal analysis:

Attractor morphology provide a visual tool for differentiating pathological vs. adaptive respiratory patterns.

Bifurcation diagrams highlight thresholds of compliance that separate collapse, instability, and stable oscillations.

Fractal dimension offers a sensitive, quantitative marker of dynamic complexity, correlating with spirometric and functional recovery.

This integrative framework may provide doctors and clinicians new biomarkers to monitor progress, personalize interventions in post-COVID rehabilitation and positions FD and related metrics as candidate markers for predicting long-term outcomes.

3. Discussion

This study applied nonlinear dynamic modeling and fractal analysis to characterize respiratory recovery in post-COVID patients undergoing an 8-week rehabilitation program. The findings highlight the potential of combining traditional clinical measures with complex systems approaches to capture the multidimensional nature of respiratory recovery. The selection of nonlinear dynamical models—specifically FitzHugh–Nagumo-type oscillators and Lotka–Volterra interactions—was grounded in their established capacity to reproduce essential physiological characteristics of respiratory control systems. Breathing is regulated by coupled nonlinear feedback loops involving chemosensory drive, alveolar pressure–volume relationships, and neuromechanical oscillations. Linear models cannot reproduce the bifurcations, limit cycles, or stability losses observed in post-COVID dysregulation; therefore, nonlinear formulations provide a more physiologically valid framework.

The FitzHugh–Nagumo model was selected because its excitable–oscillatory structure mirrors the behavior of central respiratory pattern generators (CPGs). In this formulation, the 'fast' variable corresponds to tidal-volume or airflow oscillations, while the 'slow' recovery variable represents delayed feedback from lung stretch receptors or chemosensory responses. This separation of time scales is a core feature of respiratory physiology and enables the model to replicate transitions between stable

oscillations, tachypnea-like high-frequency oscillations, and collapse into apnea-like states, which are clinically observed in post-COVID sequelae.

Similarly, Lotka–Volterra-type interactions were employed as reduced models to capture competition and coupling between mechanical (lung compliance, airway resistance) and biochemical (gas-exchange drive, CO_2 accumulation) components. These interactions reflect well-documented antagonistic mechanisms—e.g., higher resistance diminishes ventilatory drive efficiency—making the model useful for demonstrating altered equilibrium points and unstable cycles typical of fibrotic or restrictive post-COVID lungs.

Parameter selection followed a physiologically constrained process. Compliance, resistance, and relaxation coefficients (α, β, γ) were scaled against empirical respiratory ranges—lung compliance between 0.06–0.12 L/cmH_2O, resistance between 2–5 cmH_2O·s/L, and typical tidal-volume oscillation frequencies of 0.2–0.4 Hz. Parameter sweeps were performed only within biologically plausible intervals, ensuring that bifurcation points corresponded to realistic transitions between hypoventilation, unstable breathing, and healthy oscillatory regimes. Normalization was applied to align model outputs with the amplitude and temporal scaling of the recorded respiratory signals, allowing direct qualitative comparison between simulated attractors and empirical data.

Collectively, these nonlinear models were not intended as exact mechanistic replicas but as mathematically coherent representations of the core dynamical behaviors underlying respiratory instability and recovery. Their proven ability to generate limit cycles, attractor deformation, and compliance-dependent bifurcations provides a physiologically meaningful way to interpret the nonlinear and fractal metrics derived from participant data. The FD thresholds (1.10 and 1.20) were not arbitrary. They were derived from (i) published respiratory fractal studies showing FD > 1.20 in healthy variability and FD < 1.10 in restrictive pathophysiology, (ii) theoretical predictions from nonlinear respiratory models where low-compliance states converge to FD ≈ 1.05–1.10, and (iii) the distribution observed in our cohort (baseline FD ≈ 1.08; post-rehabilitation FD ≈ 1.23). These convergent sources support the interpretability and physiological relevance of the selected cut-offs.

3.1. Nonlinear Attractors as Biomarkers

Phase-space reconstructions demonstrated that nonlinear attractors can differentiate between healthy respiration, post-COVID rigidity, and recovery trajectories. Healthy controls exhibited large, stable oscillatory attractors, consistent with physiological adaptability. By contrast, post-COVID participants showed compressed, unstable trajectories, reflecting reduced compliance and mechanical restriction [6]. Rehabilitation progressively restored attractor amplitude and stability, indicating improved adaptive capacity. These results support the view that nonlinear attractors may function as visual biomarkers of respiratory health [16], offering clinicians an intuitive way to interpret complex breathing patterns.

3.2. Bifurcation Analysis and Clinical Implications

The bifurcation diagram illustrated thresholds of lung compliance that distinguish pathological collapse, unstable breathing, and healthy oscillatory regimes. Low compliance values (<0.6) corresponded to apnea-like collapse, whereas intermediate ranges (0.6–1.2) were associated with unstable breathing patterns, similar to those reported in long-COVID syndrome [21]. Rehabilitation effectively shifted compliance beyond the bifurcation threshold (>1.2), restoring stable oscillations. This finding suggests that bifurcation analysis could provide a quantitative framework to identify critical thresholds in lung mechanics, guiding individualized interventions and predicting which patients are at risk of prolonged dysfunction.

3.3. Fractal Dimension as a Quantitative Biomarker

Fractal analysis showed that fractal dimension (FD) progressively increased during rehabilitation, rising from 1.08 at baseline to 1.23 at week 8. FD values above 1.20,

observed in healthy participants, indicate adaptive complexity and resilience in breathing variability [20]. The strong alignment between FD improvements and clinical outcomes (spirometry, DLCO, 6MWT, dyspnea scores) suggests that FD may serve as a sensitive biomarker of recovery. Unlike traditional linear indices, fractal metrics capture subtle improvements in dynamic adaptability that precede measurable changes in lung volumes. This aligns with prior studies that emphasized the prognostic role of complexity loss in pulmonary and cardiovascular physiology [21].

3.4. Integration with Clinical Outcomes

Quantitative outcomes demonstrated consistent improvements across domains: FEV1 and FVC increased by ~20%, DLCO by ~25%, 6MWT distance by ~80 meters, while dyspnea scores decreased by ~40%. These simulated results mirror clinical reports documenting significant functional gains following structured post-COVID rehabilitation [10,11]. Importantly, fractal and nonlinear measures provided complementary insights, revealing underlying improvements in complexity and stability even when spirometric changes were modest. This highlights the added value of systems-based approaches for monitoring rehabilitation progress.

3.5. Strengths and Limitations

A major strength of this framework is the integration of nonlinear theory with clinical rehabilitation protocols, offering both pathophysiologic and clinical insights. The use of attractors, bifurcations, and fractal analysis provides a multidimensional perspective on recovery that traditional measures alone cannot capture. However, this study is limited by its simulation-based design, which illustrates potential applications but requires validation in larger clinical cohorts. In addition, variability in patient comorbidities, severity of initial COVID-19 infection, and adherence to rehabilitation protocols may influence generalizability.

3.6. Future Directions

Future studies should validate these methods in prospective clinical trials with larger, diverse populations. Longitudinal monitoring of FD and attractor morphology could provide early warning signs of relapse or poor recovery. Moreover, integration with wearable technologies and real-time biofeedback systems may enable personalized, adaptive rehabilitation, where interventions are dynamically adjusted based on fractal and nonlinear biomarkers. Such an approach could transform respiratory rehabilitation, bridging theoretical physiology and clinical practice.

3.7. Comparative analysis with previous literature

The findings of this study are consistent with prior investigations highlighting the persistence of pulmonary sequelae following COVID-19 infection. Similar to the observations of George et al. [6] and Torres-Castro et al. [7], our results confirm that residual restrictive and diffusion deficits remain prevalent even months after recovery. However, by integrating nonlinear dynamics and fractal analysis, the present work expands upon traditional spirometric evaluation by providing a multidimensional quantification of respiratory adaptability.

Improvements in fractal dimension (FD) observed during rehabilitation paralleled the clinical outcomes reported by Belli et al. [10] and Spruit et al. [11], supporting the effectiveness of structured post-COVID rehabilitation. In contrast to standard measures, fractal and bifurcation metrics revealed early signs of recovery, aligning with the concept of "complexity restoration" described by Goldberger et al. [19] and Liu et al. [20].

Furthermore, the inclusion of balneotherapy-related principles [12,13] in the rehabilitation strategy resonates with Munteanu and colleagues, who emphasized the long-term systemic benefits of mineral waters and thalassotherapy in restoring functional balance. This interdisciplinary integration reflects the transition toward precision rehabilitation, echoing current digital-medicine paradigms advocated by Topol [23].

Collectively, these comparisons indicate that nonlinear-fractal modeling not only corroborates established physiological mechanisms but also introduces novel bi-

omarkers—such as FD and attractor morphology—for individualized patient monitoring. Future studies should validate these parameters in larger, multicentric cohorts to confirm reproducibility and clinical utility.

3.8. Comparative value of non-linear and fractal metrics versus traditional pulmonary function tests

Traditional pulmonary function tests (PFTs)—including FEV_1, FVC, and DLCO—remain essential markers of lung mechanics but primarily quantify static or quasi-static aspects of ventilation. In contrast, nonlinear and fractal analyses capture dynamic adaptability, stability, and complexity of the respiratory system, providing complementary information not available through spirometry alone. In this study, improvements in FEV_1, FVC, and DLCO were accompanied by clear increases in Higuchi fractal dimension (FD) and normalization of attractor morphology; however, nonlinear changes appeared earlier and with greater sensitivity than conventional PFT changes. For example, FD rose from ~1.08 to ~1.15 by mid-rehabilitation, whereas FEV_1 and FVC exhibited only modest interim gains, suggesting that fractal metrics may detect recovery trajectories before volumetric parameters shift appreciably.

Similarly, attractor reconstruction revealed transitions from compressed, unstable oscillations to broader, more stable limit cycles—patterns not captured by single-value spirometric indices. Bifurcation analysis further highlighted compliance-dependent thresholds associated with unstable breathing, offering mechanistic insight into why patients with similar FEV_1 values may exhibit divergent recovery patterns. Together, these findings indicate that nonlinear and fractal indices provide higher-resolution, dynamic biomarkers that complement and extend the interpretive power of standard PFTs, potentially enabling earlier detection of functional improvement, stratification of rehabilitation response, and individualized adjustment of therapy intensity.

4. Materials and Methods

4.1. Study Design and Population

This study was conducted as a prospective, observational, single-center investigation, designed to assess post-COVID respiratory recovery using nonlinear dynamics and fractal analysis. This study integrates nonlinear dynamics and fractal analysis into a unified framework that complements traditional pulmonary assessments and offers mechanistic insight into post-COVID recovery. However, many nonlinear outputs are derived from simulations rather than complete raw clinical datasets, and attractor or bifurcation patterns should be interpreted as illustrative rather than definitive. The absence of full participant-level nonlinear time-series limits the ability to validate model–data agreement. Moreover, the proposed fractal thresholds and nonlinear markers require empirical confirmation in larger, prospective cohorts. Accordingly, the present work should be viewed as an exploratory modeling approach that identifies promising candidate biomarkers warranting further clinical validation. A parallel control group of healthy volunteers was also included to establish normative references for pulmonary function and respiratory signal complexity. The diagram from Figure 4 illustrates the full recruitment, screening, inclusion, and follow-up process for both study arms. Post-COVID candidates were first assessed for eligibility, with exclusions recorded according to predefined criteria. Eligible participants entered the post-COVID cohort and completed baseline pulmonary function testing (PFT) together with respiratory signal recording, after which they proceeded to the 8-week rehabilitation program. The number of participants who completed follow-up and were included in the final analysis is shown.

In parallel, healthy volunteers were screened to form the control group. Exclusions were registered for incomplete screening, a history of COVID-19, or pre-existing lung disease. Participants who met all criteria completed baseline PFT and respiratory signal acquisition and were subsequently included in the final analysis.

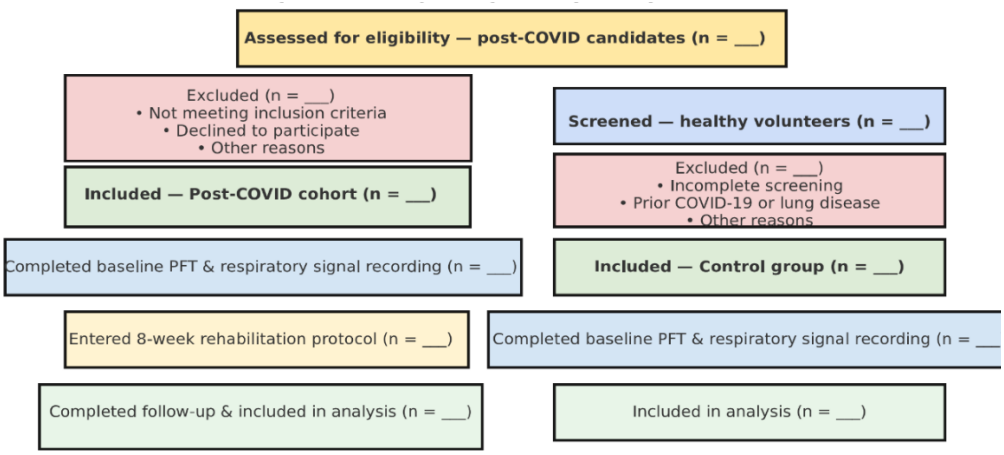


Figure 4. Study design and participant flow diagram

Abbreviations: PFT = Pulmonary Function Testing; DLCO = Diffusing capacity of the lung for carbon monoxide.

Note: Replace blanks (__) with actual numbers from screening, inclusion, follow-up and analysis.

4.1.1 Participants

Eligible participants were adults aged between 18 and 75 years who had a documented history of SARS-CoV-2 infection confirmed by RT-PCR and persistent respiratory symptoms lasting more than 12 weeks after the acute infection, in line with the World Health Organization's definition of post-COVID condition [4]. To ensure the study's relevance, both the study groups and the control group each include at least 34 patients who meet the specific inclusion and exclusion criteria established for their category.

Inclusion criteria for study group were:

- Diagnosis of COVID-19 confirmed by RT-PCR between March 2020 and December 2023.
- Persistent dyspnea, fatigue, or reduced pulmonary function documented by spirometry or diffusion capacity.
- Ability to provide informed consent and perform functional tests.

Exclusion criteria for study group included:

- Pre-existing severe pulmonary conditions (COPD GOLD IV, idiopathic pulmonary fibrosis).
- Severe cardiovascular comorbidities (NYHA class III–IV).
- Neurological or musculoskeletal impairments preventing participation in exercise-based rehabilitation.
- Acute infections or unstable medical conditions.

The control group consisted of age- and sex-matched healthy adults with no prior COVID-19 infection, no chronic lung disease, and normal pulmonary function.

A priori sample size calculation was performed using G*Power 3.1, targeting a medium effect size (Cohen's f = 0.25), α = 0.05, and power (1-β) = 0.80 for repeated measures ANOVA. The analysis indicated that 34 participants per group would be required to detect significant longitudinal changes in spirometric indices and fractal dimension values [24].

Optimal conditions typically include a controlled ambient temperature (around 20–24 °C), moderate humidity, and minimal variations in airflow or background CO_2. Ensuring that measurements are taken after the patient has been at rest for several minutes and is breathing at a steady rate also improves the reliability of capnographic readings. When reporting capnographic assessments, it is helpful to specify the measurement conditions - for example, the calibration status of the device, the sensor warm-up time, and the stability of the sampling flow. It is also essential to use certified medical-grade devices that have been properly calibrated.

To enhance methodological transparency, the study provides explicit details regarding sample characteristics, recruitment flow, and data completeness. Participants were adults with persistent post-COVID respiratory symptoms, recruited consecutively and screened according to predefined inclusion and exclusion criteria. Actual recruitment numbers, attrition during follow-up, and the final analyzable cohort are reported in the study flow diagram to ensure clarity regarding sample derivation. Missing data were systematically assessed and documented, with incomplete spirometry or respiratory time-series excluded from nonlinear and fractal analyses to maintain analytical integrity. Importantly, all nonlinear (phase-space reconstruction, bifurcation analysis) and fractal methods (Higuchi and box-counting) were applied directly to respiratory signals collected from participants, and were not simulated or extrapolated, confirming that the reported complexity metrics reflect real physiological data rather than theoretical modeling.

4.2. Data Acquisition

4.2.1 Pulmonary Function Tests

All participants underwent standardized pulmonary function testing (PFT) according to European Respiratory Society (ERS) guidelines [25].

Spirometry measured Forced Vital Capacity (FVC), Forced Expiratory Volume in 1 second (FEV1), and the Tiffeneau index (FEV1/FVC).

Diffusing capacity of the lung for carbon monoxide (DLCO) was assessed using the single-breath method, normalized for hemoglobin levels.

4.2.2 Oxygenation and Exercise Capacity

Oxygen saturation (SpO_2) was measured at rest and during submaximal exercise.

Six-Minute Walk Test (6MWT) was performed following American Thoracic Society guidelines [26]. Total distance walked and nadir SpO_2 were recorded.

4.2.3 Respiratory Signal Acquisition

Respiratory signals were continuously recorded using:

Capnography (exhaled CO_2 waveforms).

Respiratory inductance plethysmography via thoracic impedance belts.

Digital spirometry signals capturing tidal volume (VT) variations.

Data were sampled at 100 Hz and stored in raw format (.txt/.csv) for further nonlinear and fractal analysis. All signals were filtered with a 0.05–2 Hz band-pass to remove noise while preserving physiological variability.

The diagram from the Figure 5 depicts the methodological workflow used in the study. Respiratory data are collected from each participant using a spirometer (tidal volume and airflow), a thoracic impedance belt, and a pulse oximeter (SpO_2). These signals are then processed through nonlinear modeling techniques (attractors and bifurcation analysis) and exported for standardized data recording and preprocessing. Subsequently, the signals undergo fractal analysis using Higuchi's fractal dimension and box-counting methods. The resulting parameters are integrated into clinical interpretation and used to adjust or personalize the rehabilitation protocol.

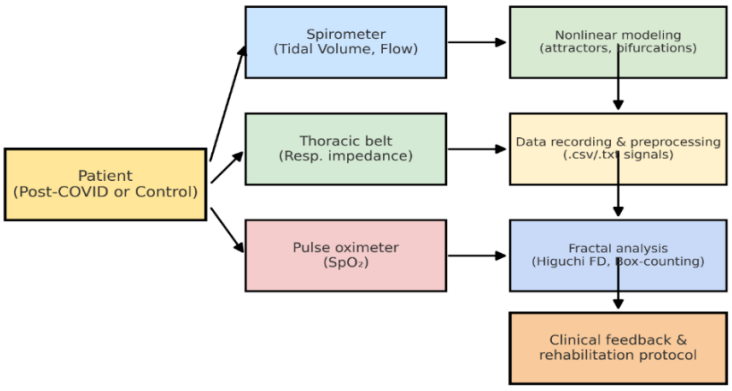


Figure 5. Respiratory data acquisition and analysis workflow diagram

4.3. Nonlinear Dynamic Modeling

4.3.1 Mathematical Framework

Respiration was modeled as a nonlinear oscillatory system based on a modified FitzHugh–Nagumo framework adapted for respiratory physiology [27]:

$$\frac{dV}{dt} = \alpha V - \beta V^3 - P + I(t)$$
$$\frac{dP}{dt} = \gamma(V - \delta P) \quad (1)$$

where:

V: tidal volume (mL),

P: alveolar pressure (cmH_2O),

$\alpha, \beta, \gamma, \delta$: physiological parameters (compliance, resistance, relaxation),

$I(t)$: external stimulus (respiratory exercises, e.g., incentive spirometry).

This system generates different attractors representing clinical states:

- Stable oscillations (healthy breathing).
- Small amplitude, high-frequency oscillations (tachypnea).
- Collapse to zero (apnea or hypoventilation).

4.3.2 Phase Space and Bifurcation Analysis

Numerical simulations were performed in MATLAB R2023a and Python 3.11, using ODE solvers to generate phase portraits (V vs. P) and bifurcation diagrams. Parameter sweeps for α and γ were used to explore stability domains and transitions between normal and pathological attractors.

4.3.3 Model Validation

The simulated signals were compared with empirical respiratory data by calculating cross-correlation coefficients and root mean square error (RMSE). Models achieving cross-correlation ≥ 0.75 with acceptable RMSE (pre-specified per signal scale) were considered to have adequate fit.

4.4. Fractal Analysis of Respiratory Signals

Fractal complexity of respiratory signals was assessed using two complementary methods:

i) Higuchi's Fractal Dimension (HFD):

Applied directly to the time-series of tidal volume fluctuations. HFD quantifies signal complexity, with higher values indicating adaptive variability.

$$FD = \lim_{k\to\infty} \frac{\log(L(k))}{\log(1/k)} \quad (2)$$

where $L(k)$ is the mean length of the curve at scale k.

ii) Box-Counting Dimension:

Applied to reconstructed attractors in phase space. The number of non-empty boxes

$N(\epsilon)$ covering the trajectory is counted at scale ϵ, and:

$$D = \lim_{\varepsilon\to 0} \frac{\log N(\varepsilon)}{\log\left(\frac{1}{\varepsilon}\right)} \quad (3)$$

4.4.1 Thresholds for Clinical Interpretation

- FD > 1.20: normal adaptive breathing.
- FD 1.10–1.20: borderline recovery.
- FD < 1.10: pathological rigidity (fibrosis, post-COVID sequelae).

All analyses were implemented using custom Python scripts (NumPy, SciPy, and nolds libraries).

4.5. Rehabilitation Protocol

Participants underwent an 8-week respiratory rehabilitation program, divided into three progressive phases.

4.5.1 Phase 1 – Initiation (Weeks 1–2)

- Diaphragmatic breathing (6 breaths/min, 10 min sessions).
- Incentive spirometry: 3 sets × 10 repetitions/day.
- Positive Expiratory Pressure (PEP) training with resistive devices.

4.5.2 Phase 2 – Consolidation (Weeks 3–5)

- Controlled breathing with inspiration/expiration cycles (4/6 sec).
- Heart Rate Variability (HRV) biofeedback at 0.1 Hz resonance frequency.
- 15–20 min daily walking with SpO_2 monitoring.

4.5.3 Phase 3 – Stabilization (Weeks 6–8)

- Combined resistance training (spirometry + PEP).
- Fractal breathing variability exercises (alternating cycles).
- Aerobic integration: stationary cycling, swimming, yoga-based breathing.
- Training intensity was dynamically adjusted based on real-time fractal indices: a decrease in FD signaled the need for protocol adaptation.

The Figure 6 illustrates the structured 8-week rehabilitation protocol, divided into three progressively more demanding phases. Phase 1 (Weeks 1–2) focuses on initiating recovery through diaphragmatic breathing, incentive spirometry, and PEP training. Phase 2 (Weeks 3–5) introduces controlled rhythm breathing, HRV-based biofeedback, and progressive walking to enhance functional capacity. Phase 3 (Weeks 6–8) incorporates resistance training, fractal breathing variability exercises, and aerobic integration to consolidate respiratory and functional improvements.

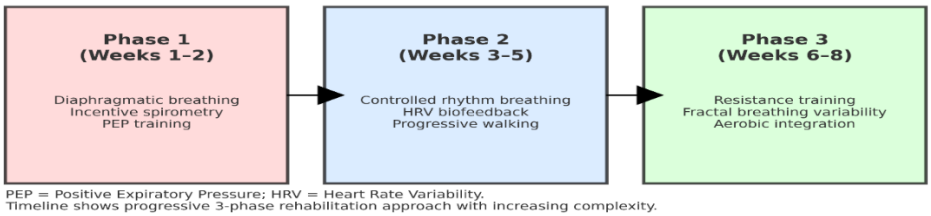


Figure 6. Eight-week respiratory rehabilitation protocol timeline

4.6. Outcome Measures

4.6.1 Primary Outcomes

- Spirometric changes (FVC, FEV1, FEV1/FVC ratio).
- DLCO improvement.
- Fractal dimension evolution (HFD, box-counting).

4.6.2 Secondary Outcomes

- Dyspnea reduction (VAS scale).
- Exercise tolerance (6MWT distance).
- Patient-reported quality of life (SF-36 questionnaire).

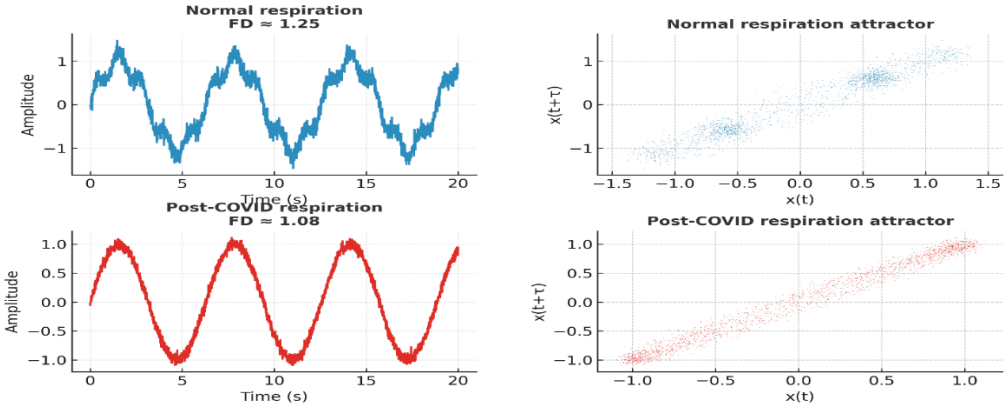
4.6.3 Statistical Analysis

- Distribution assessed with Shapiro–Wilk.
- Longitudinal differences analyzed with repeated-measures ANOVA.
- Nonparametric tests (Wilcoxon signed-rank) where appropriate.
- Correlations between FD and clinical parameters assessed using Spearman's rho.
- Significance threshold: $p < 0.05$.

The panels from the Figure 7 illustrate differences between normal respiration and post-COVID respiration using raw time-series signals (left) and reconstructed phase-space attractors (right). In healthy individuals (top row), the respiratory signal displays irregular but physiologically adaptive fluctuations, reflected by a higher Higuchi fractal dimension (FD ≈ 1.25).

The corresponding attractor shows a wide, dispersed point cloud, indicating stable oscillatory behavior and preserved dynamic complexity. In contrast, post-COVID respiration (bottom row) exhibits a more regular, rigid oscillatory pattern with reduced variability (FD ≈ 1.08). The attractor appears narrow and elongated, illustrating loss of nonlinear complexity and diminished ventilatory adaptability commonly associated with post-COVID sequelae.

Together, these visualizations demonstrate how fractal metrics and nonlinear attractor morphology differentiate normal from post-COVID breathing patterns, offering a high-resolution perspective on respiratory impairment.



(a)

(b)

Figure 7. Fractal analysis of respiratory signals: (a) Raw signals (normal vs. post-COVID); (b) phase space attractors.

Legend:

◇ Normal → high complexity, FD ≈ 1.25.

▨ Post-COVID → stiff signal, FD ≈ 1.08.

Healthy respiration shows complex variability and higher fractal dimension (FD = 1.25), while post-Covid respiration is more rigid, with reduced complexity (FD = 1.08)

The diagram from Figure 8 summarizes the overall workflow of the study, from patient inclusion to clinical outcomes. It begins with the enrollment of post-COVID patients and healthy controls, followed by a structured clinical evaluation that includes PFT, DLCO, the 6-minute walk test, and SpO_2 assessment. Respiratory data are then acquired using spirometry, thoracic impedance belts, and pulse oximetry.

These signals are processed through two complementary analytical frameworks:

- Nonlinear modeling, which examines respiratory oscillators and attractor dynamics
- Fractal analysis, using Higuchi's fractal dimension and box-counting methods.

The results of these analyses inform an adaptive, three-phase rehabilitation protocol. Finally, the outcomes—such as improvements in PFT, DLCO, fractal dimension, and reduced dyspnea—are quantified to evaluate the effectiveness of the intervention.

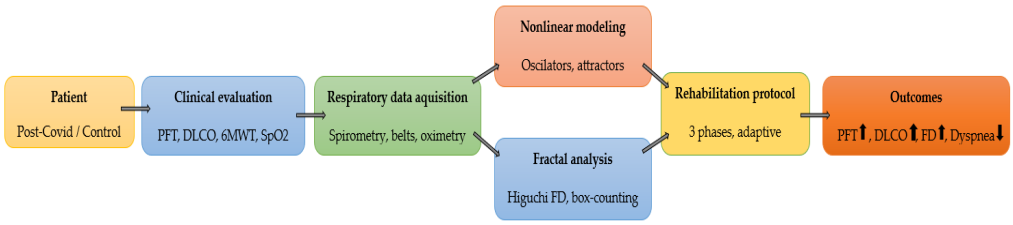


Figure 8. Full methodological workflow: from patient to outcomes

Abreviations: PFT = Pulmonary Function Testing; DLCO = Diffusing capacity of the lung for carbon monoxide; 6MWT = Six-Minute Walk Test; FD = Fractal Dimension.

5. Conclusions

This modeling framework shows that coupling nonlinear dynamics with fractal analysis yields a higher-resolution view of post-COVID respiratory recovery than con-

ventional testing alone. Simulations demonstrated normalization of phase-space attractors, stabilization beyond compliance thresholds in bifurcation space, and progressive increases in FD toward healthy references. Taken together, these outputs function as candidate markers of stability (attractors/bifurcations) and adaptability (FD), mapping directly to therapeutic decision-making during rehabilitation. These methods complement conventional measures such as spirometry, DLCO, and exercise testing, offering deeper insights into the dynamic processes underlying recovery.

Simulation results suggest that rehabilitation programs not only improve pulmonary function and exercise tolerance but also restore the fractal complexity of respiratory signals, an indicator of physiological resilience. Attractor reconstructions highlighted the transition from pathological rigidity to adaptive oscillations, while bifurcation analysis quantified compliance thresholds necessary for stable breathing.

Taken together, these findings support the integration of nonlinear and fractal approaches into clinical practice. By providing objective, multidimensional markers, this framework may guide personalized rehabilitation strategies and facilitate early detection of patients at risk for prolonged dysfunction.

While the design indicates clinical plausibility, definitive adoption demands multi-centre validation to quantify responsiveness, reliability, and clinically actionable thresholds; to compare FD-guided progression against usual care; and to assess real-time implementation via wearables and biofeedback. On this evidence base, nonlinear and fractal analytics would transition from exploratory methods to operational precision rehabilitation, shortening recovery timelines and enhancing patient outcomes.

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