

Research article

Validation of the mathematical model of upper body biomechanics using neural networks

Elena Mereuta ¹, Monica-Iuliana Novetschi ¹, Daniel Ganea ¹, Valentin Tiberiu Amortila ^{1*} and Tarek Nazer ¹

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¹ Dunarea de Jos University of Galati, Romania; emereuta@ugal.ro, monica.novetschi@ugal.ro, daniel.ganea@ugal.ro, valentin.amortila@ugal.ro, nazer_tarek@yahoo.com

* Correspondence: Valentin Amortila, valentin.amortila@ugal.ro

Abstract: The paper presents artificial neural networks (ANNs) as a tool for validating the mathematical model. The neural network architecture must learn the relationship between the inputs (which are the parameters of the biomechanical mathematical model, such as height and body mass) and the desired outputs (i.e. the perturbing forces of the biomechanical model elements). We used part of the perturbing force values of the muscle groups responsible for the spine, neck, and head movement, determined using the mathematical model and the C++ application, to train the neural network. We used the remaining data to validate the neural network. The neural network architecture was created using the Easy NN application. After training the network, we concluded that the subject's height has the most significant impact on generating muscle force and is also the most sensitive parameter. The muscle force values of the data used for validation are almost equal to those determined using the mathematical model. Therefore, we can conclude that the mathematical model is correct, and the neural network can make predictions for various subject dimensions, even if their values are not within the range of values for which we trained the network.

Keywords: mathematical model; neural networks; human body

1. Introduction

For researchers, determining the equations of motion to analyze the dynamics of the human body has been a long-standing challenge. Mathematical models employ anthropometric data and simplify assumptions to describe the movement of various body parts under external forces, which can sometimes skew the findings. When examining the biomechanical system of the head, neck, and vertebral column, experts frequently concentrate on routine or coerced movement in the humeral directions. When examining the biomechanical system of the head, neck, and vertebral column, experts frequently concentrate on routine or coerced movement in the following directions:

- Creating a 3D dynamic model of the head-neck system for the extension-flexion motion when dynamic forces generated on impact appear [1];
- Developing a mathematical model of the neck, consisting of seven rigid bodies that represent the cervical vertebrae and the upper thorax connected by joints, to which we added two muscle substitutes with elastic and damping properties which present a dynamic response similar to the natural response to the impact of a human subject [2];
- Developing biomechanical models that study the response of the head-neck structure to impact, considering dynamic loading as input. The associated mathematical models follow the predictions of the system's movement in specific initial conditions [3].
- A systematic classification of the cervical spine models highlighted the following trends: multibody modelling for the simulation of impact situations, inverse

dynamics, optimization, and the Hill muscle model. To these, the models aimed at validating the results through comparisons with the data from the literature are added, most of them not being able to confirm the similarity with the actual model [4]

- Mathematical modelling was used to determine how an end effector attached to the human upper limb moves. The method uses rotation matrices to determine the general transfer matrix, called the homogeneous transformation matrix [5].
- Other researchers developed a detailed finite element model (FEM) of the human neck. They developed a biomechanical subsystem by coupling the head and validating the model with kinematic data collected from subjects [6].
- The dynamic behaviour of the human head and neck in car crashes was studied using a detailed three-dimensional mathematical model implemented in the integrated multibody/finite element package [7].

Traditional methods used in biomechanical modelling usually involve formulating and solving differential equations that describe the studied system's behaviour. These equations can be highly complex, depending on the system's nature and the study conditions. An innovative approach in biomechanical modelling involves the use of artificial neural networks (ANNs) that we can use to approximate relationships between system inputs (such as the initial configuration of the body) and relevant output data (such as muscle forces, variations angular or displacements of anatomical elements).

2. Method

Developing the mathematical model in biomechanics involves studying the specialized literature, collecting data, choosing parameters and variables, and calibrating the model based on anthropometric data. The head, neck, and spine subsystem is a complex biomechanical system responsible for supporting and controlling the movement of the head, neck, and trunk. A mathematical model was developed to understand this system better, considering it a triple-inverted physical pendulum [8]. The model consists of articulated bars for the spine and neck and a sphere for the head anatomical element. This model has three degrees of freedom, and three disturbing forces act on it: F_1 , F_2 , and F_3 . These forces generate motion and alter the orthostatic position, which is the natural upright position of the body (Figure 1). This mathematical model can provide analytical data regarding the forces responsible for the head, neck, or trunk movement and the restorative forces responsible for returning the body to the orthostatic position. The analytical expressions of the forces generated by the muscle groups for the movement of the three anatomical elements are [8]:

$$\left[\frac{m_1 \cdot L_{cv}^2}{3} + 2 \cdot m_2 \cdot L_{cv}^2 + m_3 \cdot L_{cv}^2 + m_2 \cdot L_{cv} \cdot a_2 \cdot \cos(\theta_2 - \theta_1) + m_3 \cdot L_{cv} \cdot L_g \cdot \cos(\theta_2 - \theta_1) + \frac{1}{2} \cdot m_3 \cdot L_{cv} \cdot a_3 \cdot \cos(\theta_3 - \theta_1) \right] \cdot \ddot{\theta}_1 + (m_1 \cdot g \cdot a_1 + m_2 \cdot g \cdot L_{cv} + m_3 \cdot g \cdot L_{cv} + 2 \cdot k_1 \cdot a_1^2 + 2 \cdot k_2 \cdot L_{cv}^2 + 4 \cdot k_3 \cdot L_{cv}^2 + 2 \cdot k_2 \cdot a_2 \cdot L_{cv} + 4 \cdot k_3 \cdot L_g \cdot L_{cv} + 2 \cdot k_3 \cdot a_3 \cdot L_{cv}) \cdot \theta_1 = F_1 \quad (1)$$

$$[m_2 \cdot L_{cv} \cdot a_2 \cdot \cos(\theta_2 - \theta_1) + m_3 \cdot L_{cv} \cdot L_g \cdot \cos(\theta_2 - \theta_1) + \frac{m_2 \cdot L_g^2}{12} + m_2 a_2 + m_3 \cdot L_g^2 + \frac{1}{2} \cdot m_3 \cdot L_g \cdot a_3 \cdot \cos(\theta_3 - \theta_2)] \cdot \ddot{\theta}_2 + (2 \cdot k_2 \cdot a_2 \cdot L_{cv} + 4 \cdot k_3 \cdot L_g \cdot L_{cv} + m_2 \cdot g \cdot a_2 + m_3 \cdot g \cdot L_g + 2 \cdot k_2 \cdot a_2^2 + 2 \cdot k_3 \cdot L_g^2 + 2 \cdot k_3 \cdot a_3 \cdot L_g) \cdot \theta_2 = F_2 \quad (2)$$

$$\begin{aligned} & \left[\frac{1}{2} \cdot m_3 \cdot L_{cv} \cdot a_3 \cdot \cos(\theta_3 - \theta_1) + \frac{1}{2} \cdot m_3 \cdot L_g \cdot a_3 \cdot \cos(\theta_3 - \theta_2) + \frac{1}{5} m_3 \cdot L_c^2 + m_3 \right. \\ & \quad \cdot a_3^2 \cdot \ddot{\theta}_3 + (2 \cdot k_3 \cdot a_3 \cdot L_{cv} + 2 \cdot k_3 \cdot a_3 \cdot L_g + m_3 \cdot g \cdot a_3 + 2 \cdot k_3 \\ & \quad \cdot a_3^2) \cdot \theta_3 = F_3 \end{aligned} \quad (3)$$

Where the parameters are according to Table 1.

Table 1. The parameters of the head, neck, and spine subsystem

Anatomical element	Length	The distance from the proximal joint of the centre of gravity	Percentage of total mass
Spine	$L_{cv} = 0.288H$	$a_1 = 0.5L_{cv}$	$m_1 = 35.5\%$
Neck	$L_g = 0.052H$	$a_2 = 0.5L_g$	$m_2 = 1.5\%$
Head	$L_c = 0.130H$	$a_3 = 0.5L_c$	$m_3 = 6.6\%$

To solve the system of equations, we developed a computer application in the C++ language. This program helped us determine the maximum and minimum values of disturbing forces for 16 subjects. We obtained the anthropometric dimensions of these subjects with their consent, respecting their privacy and autonomy, according to Table 2.

Table 2 shows the maximum and minimum values of the forces of the muscle groups responsible for the movement of three elements, namely the spine, neck, and head. We calculated these values with the help of a mathematical model and the C++ application. The solution of the system of equations required the realization of a computer application in the C++ language, following which the maximum and minimum values of the disturbing forces were determined for 16 subjects whose anthropometric dimensions [9] were measured with their consent, according to Table 2.

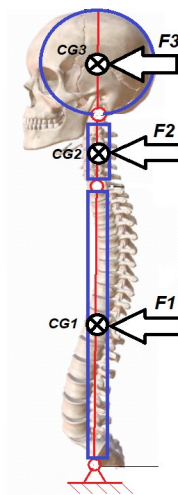


Figure 1. The disturbing forces responsible for generating the movement of the upper part of the human body [8].

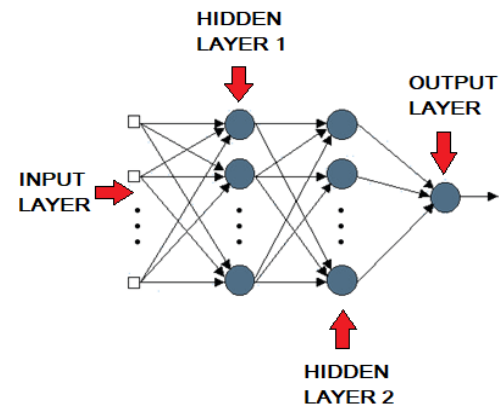


Figure 2 The general organisation of neural network architectures

Table 2. The maximum and minimum values of the forces of the muscle groups responsible for the movement of the three elements (spine, neck, head) calculated using the mathematical model and the C++ application

Height [cm]	Body mass [kg]	F1 [N]		F2 [N]		F3 [N]	
		Max	Min	Max	Min	Max	Min
187	95	18995.5	422.122	1645.52	36.5671	878.697	19.5266
182	130	18020.2	400.448	1558.7	34.6378	832.336	18.4964

165	74	14782.5	328.5	1281.11	28.4692	684.107	15.2024
168	90	15334.6	340.769	1328.12	29.5138	709.21	15.7602
179	87	17402.2	386.715	1507.74	33.5053	805.123	17.8916
172	78	16064	356.977	1392.12	30.936	743.383	16.5196
185	93	18590.6	413.125	1610.51	35.7891	860.002	19.1111
193	85	20224.1	449.424	1752.81	38.9513	935.989	20.7997
193	79	20219.5	449.323	1752.81	38.9513	935.989	20.7997
189	92	19401	431.133	1680.91	37.3535	897.593	19.9465
186	80	18782.3	417.384	1627.97	36.1771	869.324	19.3183
170	65	15684.5	348.543	1359.93	30.2207	726.196	16.1377
181	105	17805.4	395.675	1541.62	34.2582	823.215	18.2937
178	76	17200.9	382.243	1490.94	33.132	796.152	17.6923
168	58	15313.5	340.299	1328.12	29.5138	709.21	15.7602
178	68	17195.3	382.118	1490.94	33.132	796.152	17.6923

It is crucial to evaluate the performance and generalizability of a mathematical model to ensure its robustness and accuracy in making precise predictions on new data. In certain situations, applying neural networks can be an effective method for validating mathematical models. However, to validate a mathematical model using neural networks [10], [11] one must comprehensively understand the problem and the associated mathematics. Additionally, proper utilization and interpretation of neural network tools are essential.

Artificial neural networks are computing systems that can identify and replicate complex connections between inputs and outputs, similar to the connections established in the human brain. These networks consist of elementary computing units called neurons or "perceptrons". The neurons interconnected in different architectures transmit and modify information according to the intended purpose.

The architecture of neural networks can vary depending on the problem's nature and the data's complexity. In terms of biomechanical modelling, the architecture of a neural network can adapt to learn and model complex relationships between inputs and outputs. Generally, a neural network can have multiple hidden layers to learn and represent complex relationships in the input data. However, too many layers or neurons can lead to overlap and overlearning problems. The number of neurons in each layer can vary and be adjusted depending on the data's complexity and the problem's size. One such architecture is the "feed-forward MLP", where perceptrons are arranged in multiple layers, allowing information to flow in only one direction, from input to output. Each perceptron receives information from the previous perceptrons and passes it on to the perceptrons in the next layer (Figure 2).

An architecture similar to that used by the Easy NN [12], [13] application was used to validate the upper body biomechanics mathematical model. EasyNN, short for Easy Neural Network, is a software designed to create, train, and test artificial neural networks and offers an intuitive graphical interface to configure neural networks using drag-and-drop elements or context menus. The EasyNN application can generate a variety of neural network architectures, including fully connected neural networks, that allow the network to be trained automatically by selecting the training parameters (Figure 3).

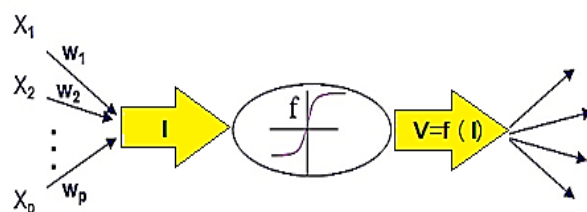


Figure 3 The direction of information flow

The neural network's output data is connected to the input data (X_p) through a set of "synaptic weights" (W_p) (Figure 3). By modifying the distribution of these weights, the network becomes less susceptible to errors that could cause certain perceptrons to be blocked. Figure 3 illustrates this relationship. Activation functions introduce non-linearities in the neural network and allow the model to learn complex relationships between input and output data. Standard activation functions include sigmoid, hyperbolic tangent, rectified linear (ReLU), and variants. The choice of activation function may depend on the problem's nature and the network's learning requirements, but the Easy NN software only uses the sigmoid function, the most used transfer function (4).

$$y = \frac{1}{1+e^{-u}} \quad (4)$$

The data set used for training and testing the neural network comes from the simulation of the mathematical model (Table 2). A single hidden layer neural network architecture was chosen for which the input data were the height and body mass of the 16 subjects. As output data, we chose the maximum and minimum values of the forces responsible for removing the upper part of the body from the orthostatic position (Figure 4).

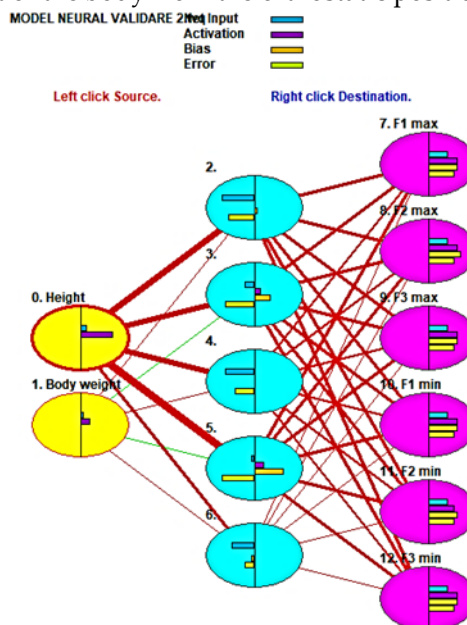


Figure 4 Architecture of the neural model used to validate the mathematical model

In practice, adjusting and optimizing neural network architecture can be an iterative process that involves experimenting with different configurations and evaluating the performance of the resulting model. We chose the single hidden layer neural network based on the least number of learning cycles and the slightest error. Neural networks with two or three hidden layers provided more cycles or a higher error.

3. Results and discussions

We used the data generated by the mathematical model for 11 subjects to train the neural network. The choice of training data was made based on the full coverage of the boundaries of the input data so that the validation data is included in this range to avoid undertraining or overtraining the network. The training of the network causes the synaptic weights to be adjusted so that the error between the provided result and the indicated value is within the prescribed limits (Figure 5).

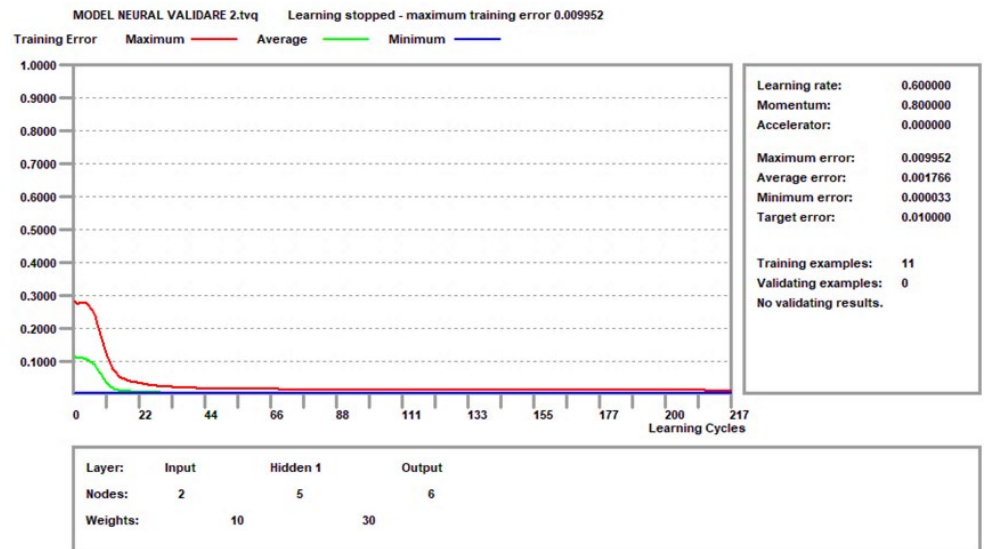


Figure 5 Neural network training graph

We conducted a model training and performance evaluation using the validation data set. In the evaluation process, we used the model to make predictions on the validation data and compared those predictions with the actual values derived from the mathematical model. After 217 training cycles, we observed a maximum error value of 2.633% and a minimum error value of 0.068% (Figure 6). Notably, the maximum errors were noticed in subjects with extreme height values at the edges of the range analyzed with the neural model. Based on these results, we conclude that the neural network generalizes effectively, with a maximum allowable error of 2.633%. Further, we confirm that the model's performance was neither over-trained nor under-trained, thanks to the adequate input data.

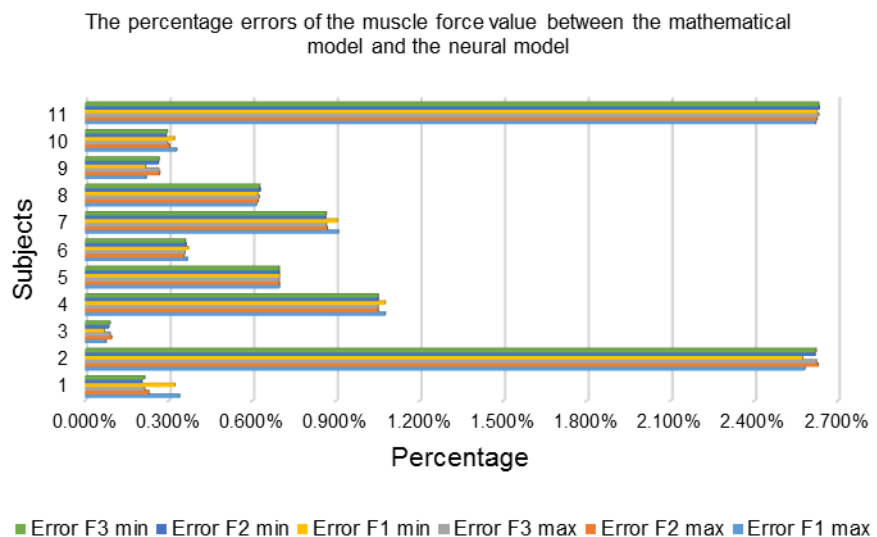


Figure 6 Errors between muscle forces determined with the mathematical model compared to the neural model

The Pearson correlation in biomechanics assessed the relationship between biomechanical variables, such as muscle strength, anthropometric data, or joint angles. This statistical method calculates a correlation coefficient, r , which can take values between -1 and +1. A perfect correlation is when this coefficient has extreme values. As the coefficient approaches 0, the correlation decreases. Thus, a direct correlation has positive values (both variables increase or both variables decrease), and an inverse correlation has negative values (one variable increases and the other decreases).

We conducted a descriptive statistical analysis using the Pearson correlation coefficient to determine which dimensions of the analyzed subjects are more significant in developing larger forces. We found that there is a correlation between the height and body mass of the subjects and the forces they develop. Based on the graph in Figure 7, we observed that all three forces have a strong correlation with the mass of the subjects (greater than 0.9) and a moderate correlation with their height (less than 0.4).

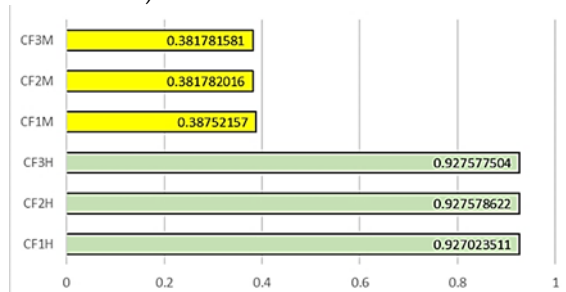


Figure 7 Correlation between forces, mass and height

To conclude, while height can affect a person's muscle strength to some extent, it is crucial to recognize that other variables can also impact muscle strength. Each individual is unique and will respond differently to stress and other factors that affect muscle strength.

The neural network has confirmed that subjects' height is important in determining muscle strength. This data gives us two essential pieces of information that help us evaluate the performance and behaviour of the network:

- Importance: This tells us how much each input affects the final result of the network. In this case, the height and body mass are essential features of the input data. Figure 8 shows that height has the most significant impact on muscle forces.

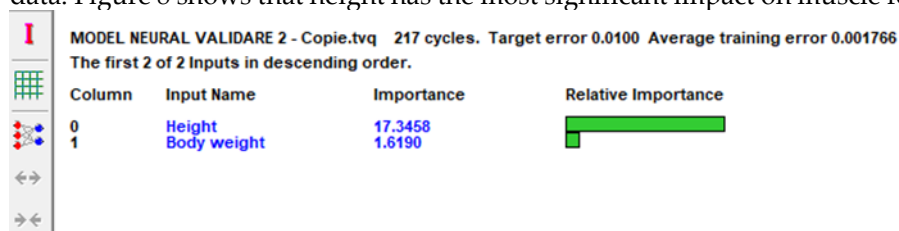


Figure 8 Importance of input data on muscle force values

- Sensitivity: This refers to how much changes in the network's inputs or parameters affect its output or prediction. A sensitive neural network reacts significantly to small changes in its input data or parameters. The height of the subjects is the most critical sensitivity in the robustness of the network and the identification of critical areas, as shown in Figure 9.

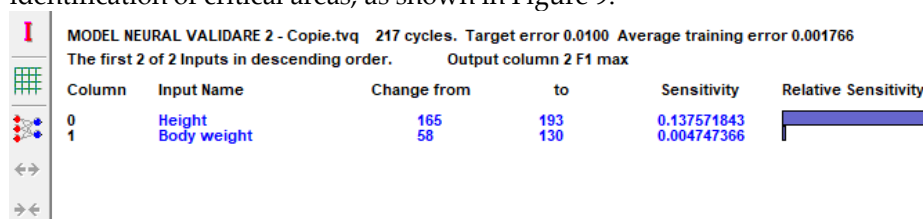


Figure 9 Sensitivity of input data on muscle force values

When testing the neural network, we used separate data from the training data. We tested the network's predictions with data from 5 subjects that we did not use in training. This data was in the same format as the training data, and Figure 10 shows how we used the query module (Query) to input this data.

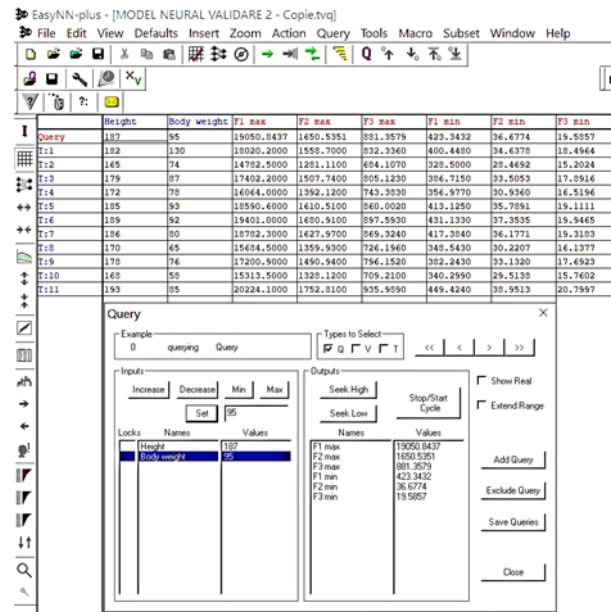


Figure 10 Using the query module (Query) for data not used when training the network

Finally, we compared the maximum forces generated by the muscle groups using the mathematical model to the neural model generated by Easy NN. We presented the maximum values obtained in testing the neural model in Table 3, and the maximum error of the maximum forces obtained in testing the neural model was less than the maximum error obtained in training the network (2.571).

Table 3 The maximum values of the muscle forces determined with the help of the mathematical model and the neural model

Height	Body weight	SUBJECTS	F1 max	
			Mathematical Model	Easy NN
187	95	SUBJECT 12	18995.5	19050.84373
168	90	SUBJECT 13	15334.6	15404.06584
193	79	SUBJECT 14	20219.5	19706.01141
181	105	SUBJECT 15	17805.4	17818.56001
178	68	SUBJECT 16	17195.3	17177.74858
Height	Body weight	SUBJECTS	F2 max	
			Mathematical Model	Easy NN
187	95	SUBJECT 12	1645.52	1650.535095
168	90	SUBJECT 13	1328.12	1335.505394
193	79	SUBJECT 14	1752.81	1707.836782
181	105	SUBJECT 15	1541.62	1543.219543
178	68	SUBJECT 16	1490.94	1488.318602
Height	Body weight	SUBJECTS	F3 max	
			Mathematical Model	Easy NN
187	95	SUBJECT 12	878.697	881.357905
168	90	SUBJECT 13	709.21	713.140823
193	79	SUBJECT 14	935.989	911.923384
181	105	SUBJECT 15	823.215	824.147061
178	68	SUBJECT 16	796.152	794.750686

5. Conclusions

The results from network testing can be compared with the performance obtained on the validation data to determine the effectiveness of the network. The accuracy of neural network predictions depends on several factors, such as the training data's quality, the neural network's size and architecture, the training algorithm, and its parameters. The presented neural model has an excellent performance with an error of up to 2.633%. However, improving the model's performance requires collecting more anthropometric data from various typologies and adjusting the model architecture or other aspects of the training process.

Using a neural network to validate the mathematical model for determining the perturbing muscle forces responsible for the movement of the human upper body offers the advantage of more easily managing the complex relationships between the data used. Comparing the RNA results with the mathematical model concludes that the neural model accurately predicts the muscle forces, validating the mathematical model. The model allows iterating on various anthropometrics as many cycles as needed to achieve a more precise analysis. Additionally, changing the parameters of the biomechanical system can establish the extreme values of the variables that correspond to anatomical limit positions.

The use of a neural network to determine the forces responsible for the movement of the upper body holds significant practical applicability in the field of auto-vehicles. It can provide the force necessary to tighten the seat belts, ensuring that in the event of an impact, the vehicle's occupants are protected from muscle damage or other bodily injuries. This not only enhances safety but also offers an economical solution, as NCAP (New Car Assessment Program) tests, which are costly, can be replaced by this efficient method.

Therefore, testing the data using a neural network is a crucial step in developing and evaluating the RNA model and plays an important role in the generalization of the prediction model. The Easy NN software allows the export of the trained neural model for use with other applications or integration into other systems, providing an advantage.

With this type of research, we have the potential to continue making significant contributions to the scientific community and enhance people's quality of life. By expanding the number of subjects to cover a broader range of anthropometric dimensions and utilizing software that allows various neural network activation functions, we can further advance our understanding and application of this technology, inspiring further innovation and progress.

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